

Reflections on F-theory and the Swampland Cobordism Conjecture

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Based On:

Hopefully Soon:

hep-th/?????.????? w/ Debray, Dierigl, Montero

hep-th/?????.????? w/ Debray, Dierigl, Montero

hep-th/?????.????? w/ Dierigl, Montero, Torres

Earlier Work:

hep-th/2107.14227 w/ Debray, Dierigl, Montero

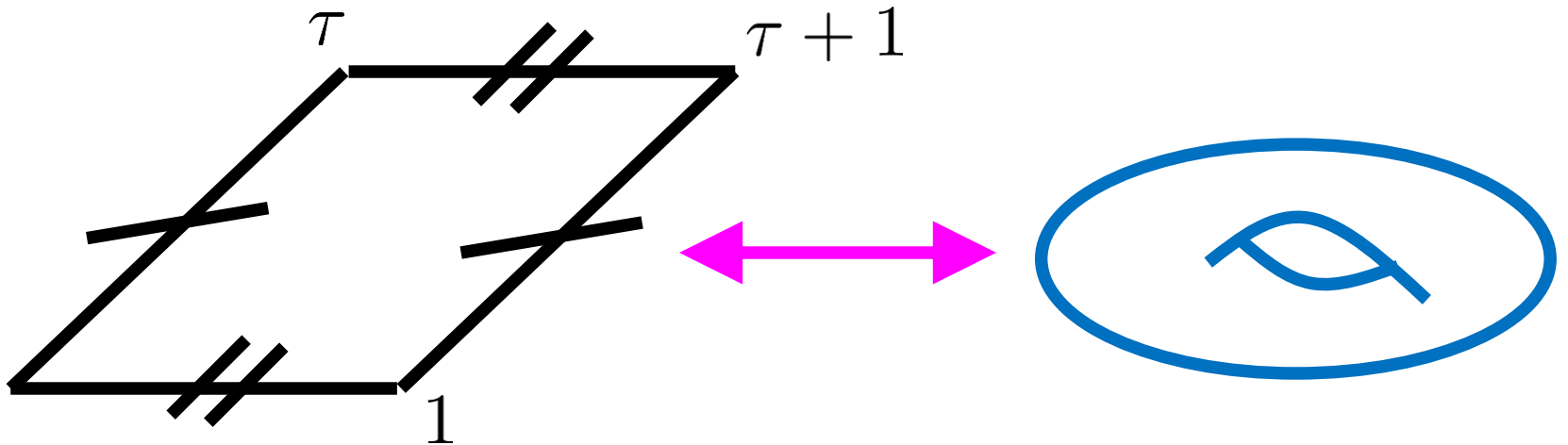
hep-th/2012.00013 w/ Dierigl

F-Theory

What is F-Theory?

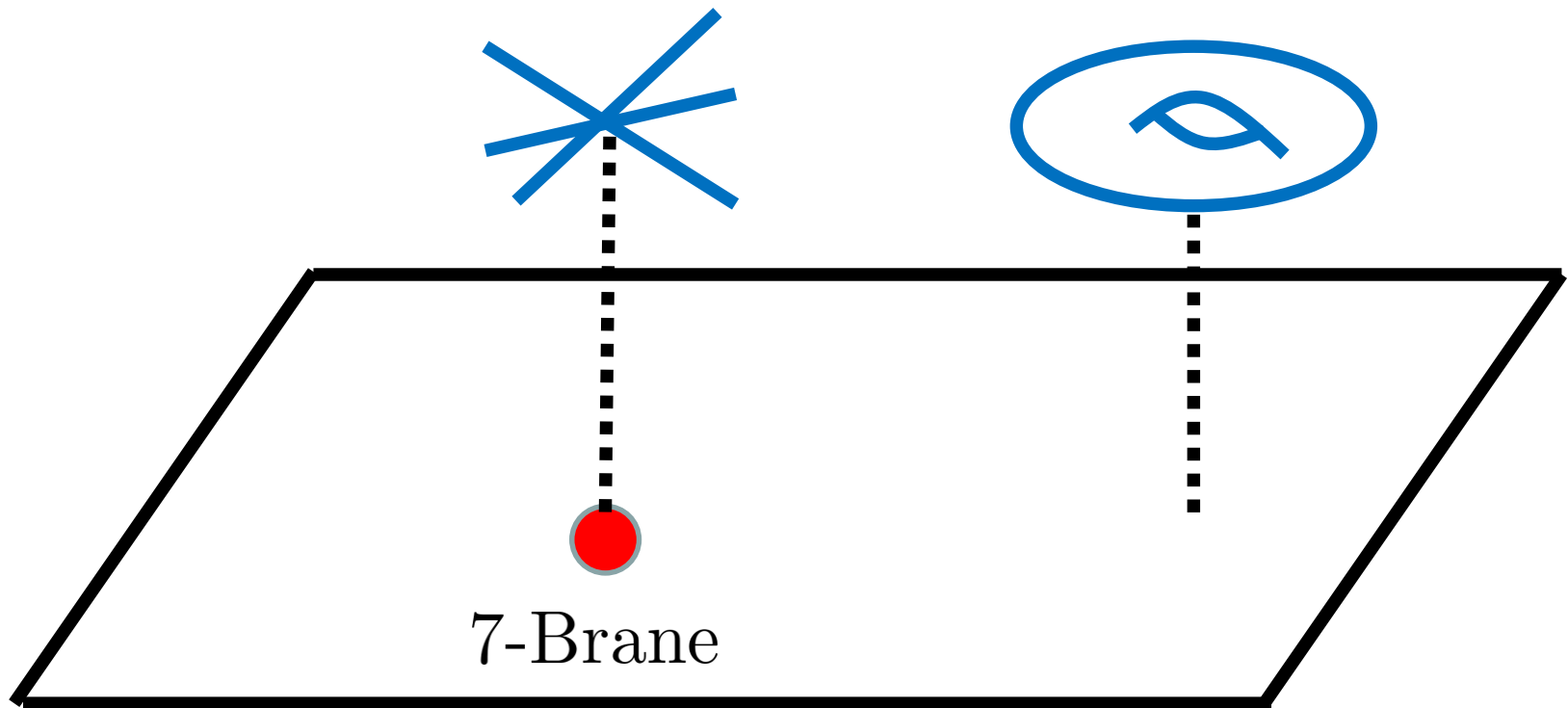
IIB Duality Implemented *Geometrically*

$$\tau = C_0 + ie^{-\Phi}$$



¿What is F-Theory?

IIB Duality Implemented *Geometrically*



F-Theory and String Feno

GUT Models: Flavor, Extra Sectors, Cosmology,...

JJH, Vafa + many

Example Feno: W-Mass Anomaly via probe D3

JJH '22 (see also Basiouris Leontaris '22)

Recently: Much Progress in Global Models

Cvetic et al.; Taylor et al.

Ingredients (Usually SUSY)

All these constructions involve \cap 7-Branes,
D3-Branes, Fluxes,... (Typically SUSY)

¿Are There Other Objects in F-Theory?

¿Non-SUSY Tools?

Non-SUSY Tools

- No superpartners have been seen (yet)
“...before the rooster crows, you will deny me three times...”
- SUSY breaking ingredients?
- How to Robustly Identify?

Swampland Cobordism Conjecture

McNamara Vafa '19

See also talk by Blumenhagen

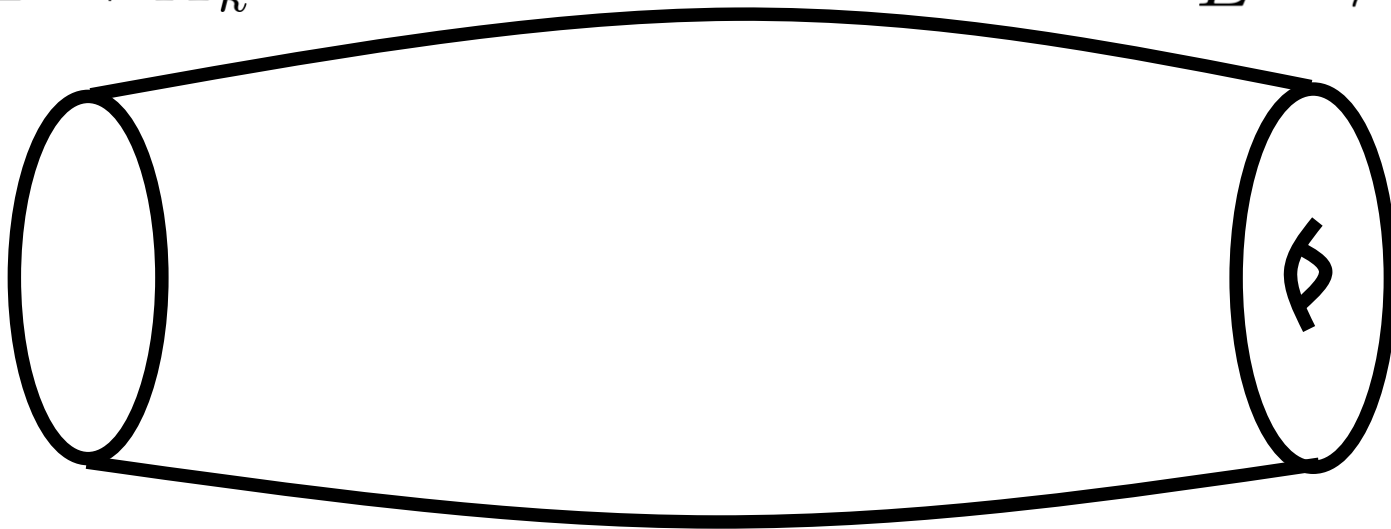
Cobordisms

Specify bundle over k -manifold: can we interpolate?

Equivalence Relation:

$$E \rightarrow X_k$$

$$E' \rightarrow X'_k$$



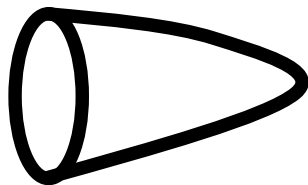
Cobordism Group: $\Omega_k^{\mathcal{G}}$

Addition:



Subtraction: Addition w/ Orientation Reversal

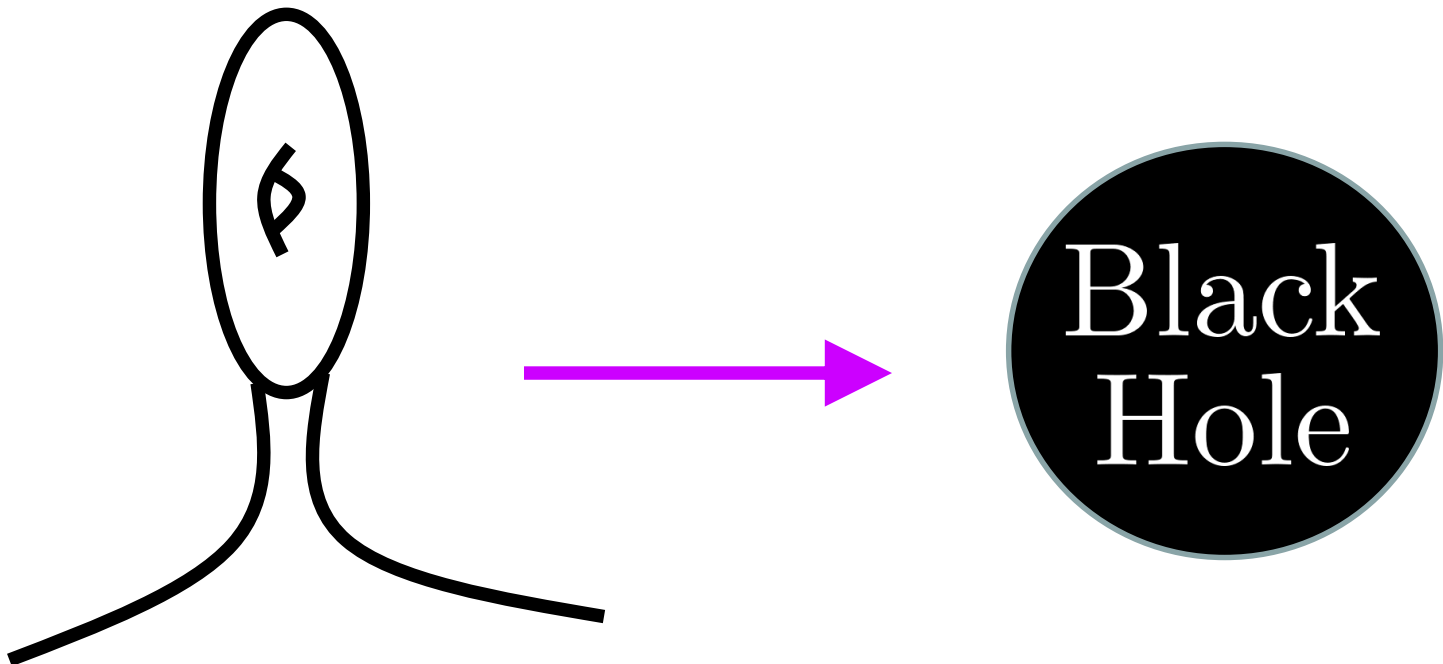
Identity:



No Global Symmetries

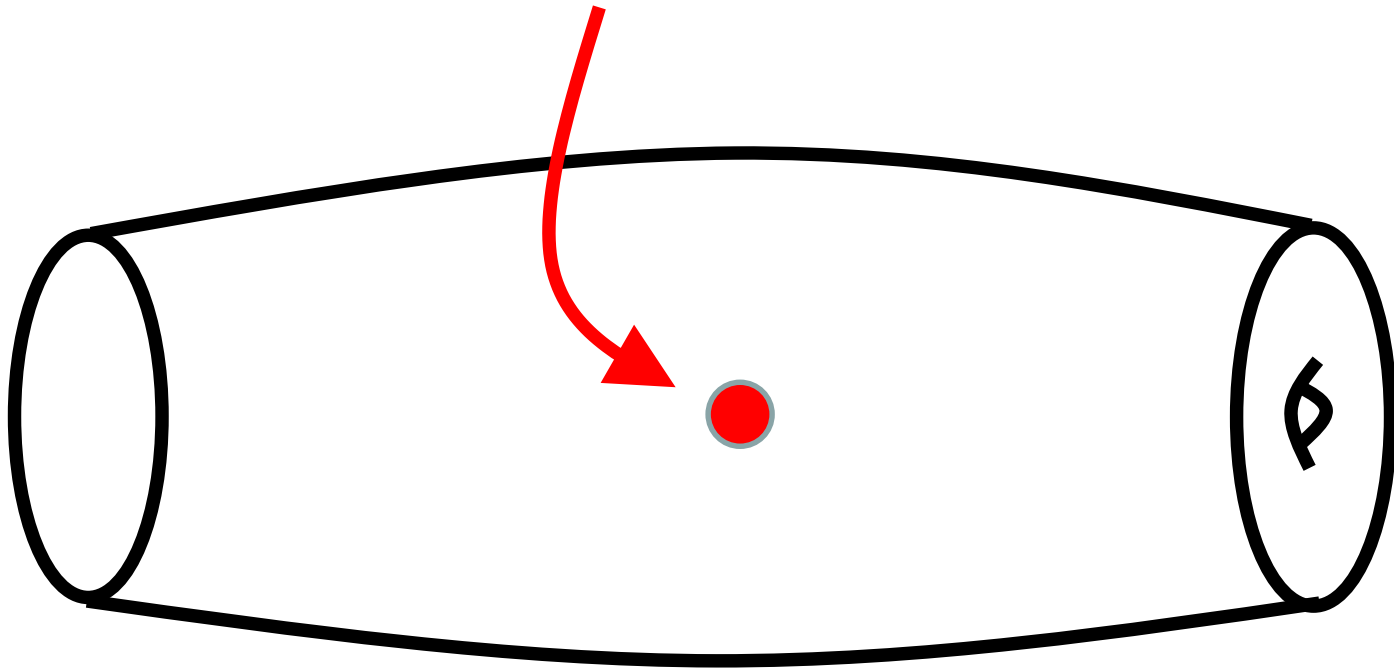
Send Topological Defect Into Black Hole...

$$\Rightarrow \Omega_k^{\text{QG}} = 0$$



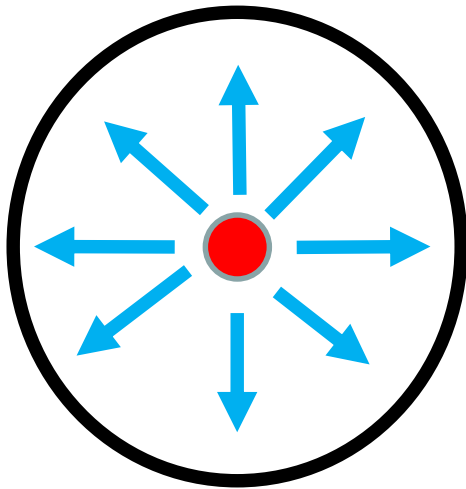
What If $\Omega_k^{\mathcal{G}} \neq 0$?

Prediction: A New Object, which trivializes Ω_k^{QG} !

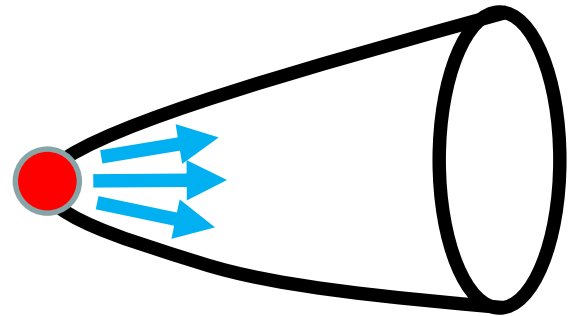


Defect View

“Flux” from Defect Measured by Bounding Space:



“Bird’s Eye View”



“In Profile”

Dualities and Bordisms

Compute $\Omega_k^{\text{IIB Dualities}}$ to predict new objects

Duality Group

IIB Duality Group

IIB SUGRA has $SL(2, \mathbb{R})$

“Broken to $SL(2, \mathbb{Z})$ by branes / flux quantizⁿ, etc.”

Including Fermions: $Mp(2, \mathbb{Z}) \rightarrow SL(2, \mathbb{Z})$
(dilatin + gravitini)

Pantev Sharpe '16

Including Reflections: Pin^+ Cover of $GL(2, \mathbb{Z}) \equiv GL^+(2, \mathbb{Z})$

Tachikawa Yonekura '18

IIB Duality Group

IIB SUGRA has $SL(2, \mathbb{R})$

“Broken to $SL(2, \mathbb{Z})$ by branes / flux quantizⁿ, etc.”

Including Fermions: $Mp(2, \mathbb{Z}) \rightarrow SL(2, \mathbb{Z})$
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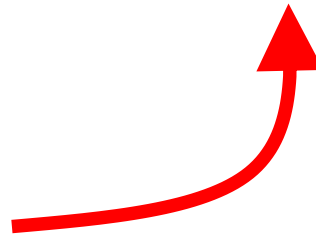
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Including Reflections: Pin^+ Cover of $GL(2, \mathbb{Z}) \equiv GL^+(2, \mathbb{Z})$
Tachikawa Yonekura '18

IIB Duality Group

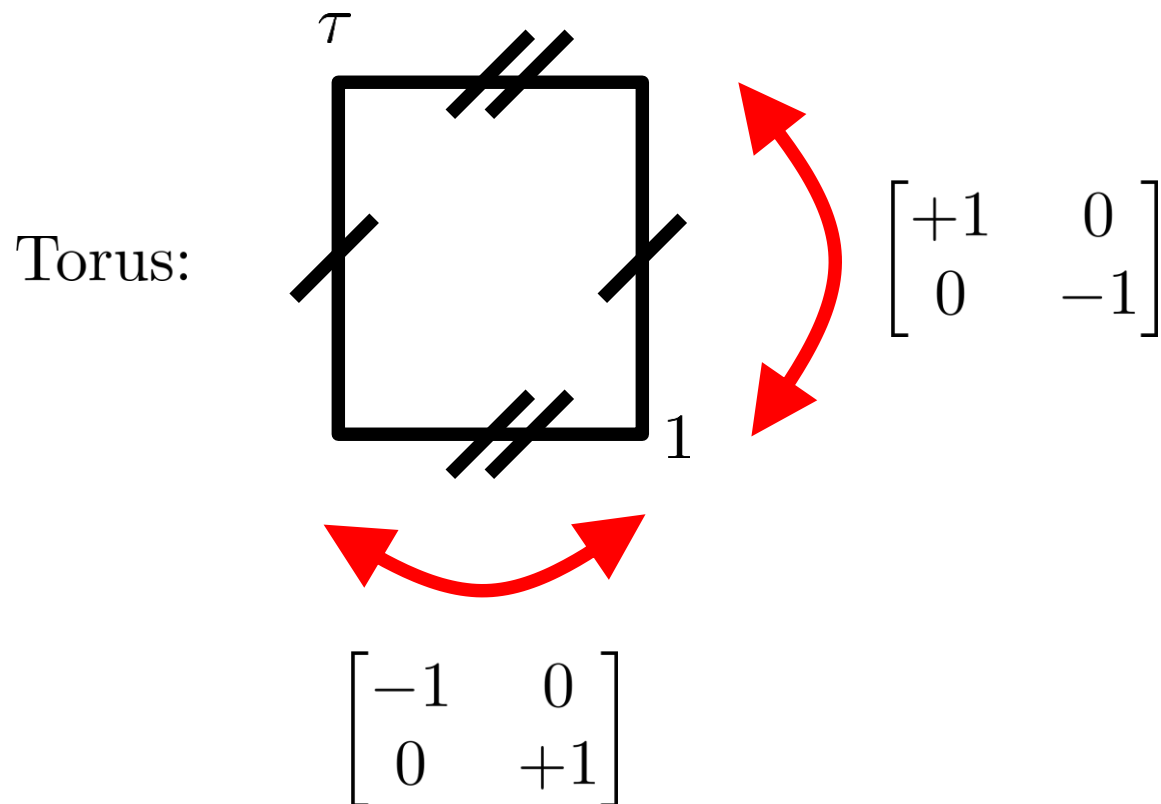
$$\begin{array}{ccc} SL(2, \mathbb{Z}) & \longrightarrow & Mp(2, \mathbb{Z}) \\ \downarrow & & \downarrow \\ GL(2, \mathbb{Z}) & \longrightarrow & GL^+(2, \mathbb{Z}) \end{array}$$

Full IIB Duality Group

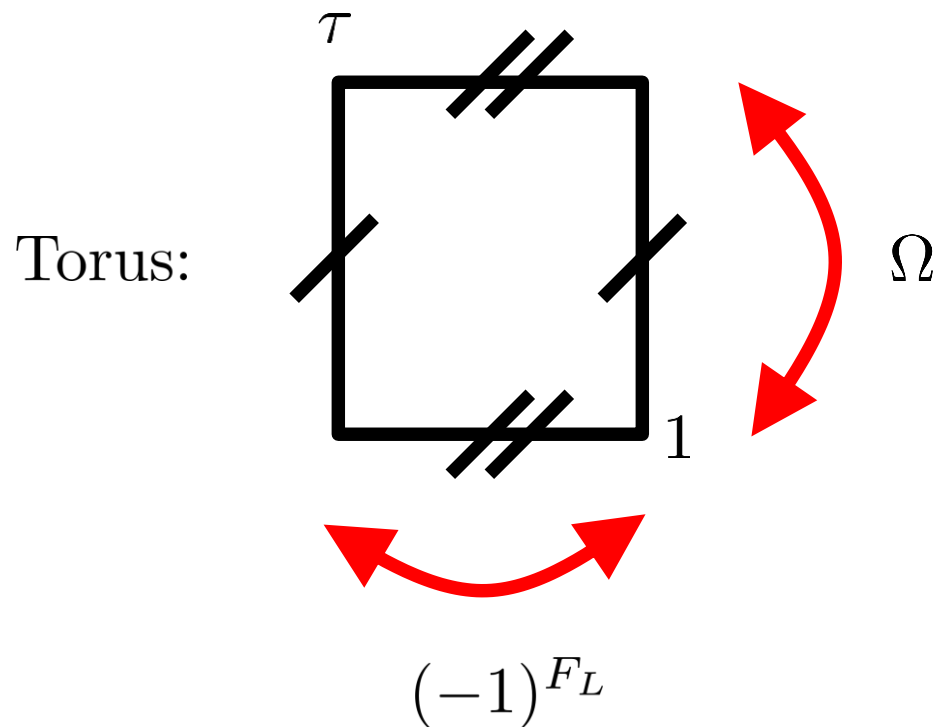


Reflections and $GL(2, \mathbb{Z})$

$$\tau = C_0 + ie^{-\Phi}: \tau \mapsto -\bar{\tau}$$



Reflections and $GL(2, \mathbb{Z})$



Bordism Duality Groups

Setup

Take IIB SUGRA (10D Spacetime) + “ \mathcal{G} -Duality Bundle”

Demand $\frac{Spin \times \mathcal{G}}{\mathbb{Z}_2}$ Structure Group

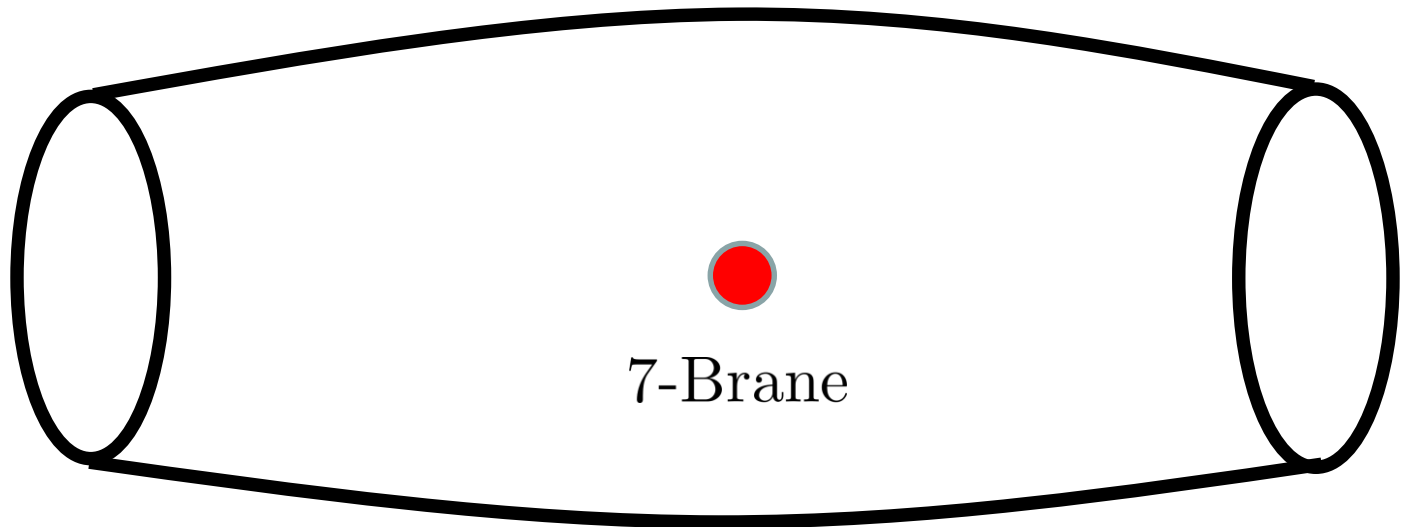
$\Omega_k^{SL(2, \mathbb{Z})}$ (Basic Duality Group)

$\Omega_k^{Mp(2, \mathbb{Z})}$ (+ Fermions)

$\Omega_k^{GL^+(2, \mathbb{Z})}$ (+ Fermions + Reflections)

Illustrative Example: $\Omega_1^{\mathcal{G}}$

Dierigl JJH '20; Debray Dierigl JJH Montero '22



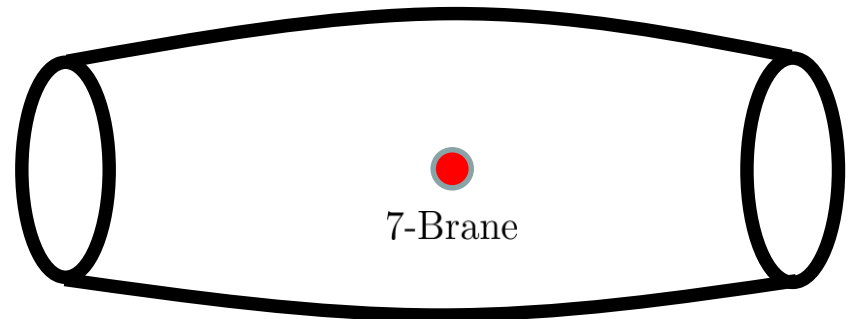
Illustrative Example: $\Omega_1^{\mathcal{G}}$

Dierigl JJH '20; Debray Dierigl JJH Montero '22

$$\Omega_1^{SL(2,\mathbb{Z})} = \mathbb{Z}_2 \times \mathbb{Z}_{12}$$

$$\Omega_1^{Mp(2,\mathbb{Z})} = \mathbb{Z}_3 \times \mathbb{Z}_8$$

$$\Omega_1^{GL^+(2,\mathbb{Z})} = \mathbb{Z}_2 \times \mathbb{Z}_2$$



$$\Omega_1^{Mp(2,\mathbb{Z})}$$

\mathbb{Z}_3 Factor of $\Omega_1^{Mp(2,\mathbb{Z})}$

$$\Omega_1^{Mp(2,\mathbb{Z})} = \mathbb{Z}_3 \times \mathbb{Z}_8$$

F-theory Picture: $\partial(T^2 \times \mathbb{C})/\mathbb{Z}_3$

Duality Bundle: $\mathbb{E} \rightarrow S^1/\mathbb{Z}_3$

These are 7-branes with $\tau = \exp(2\pi i/3)$

Example: E_6 gauge group: $y^2 = x^3 + z^4$ “Type IV^* ”

Example: $SU(3)$ gauge group: $y^2 = x^3 + z^2$ “Type IV ”

\mathbb{Z}_8 Factor of $\Omega_1^{Mp(2,\mathbb{Z})}$

$$\Omega_1^{Mp(2,\mathbb{Z})} = \mathbb{Z}_3 \times \mathbb{Z}_8$$

F-theory Picture: $\partial(T^2 \times \mathbb{C})/\mathbb{Z}_4$

Duality Bundle: $\mathbb{E} \rightarrow S^1/\mathbb{Z}_4$

These are 7-branes with $\tau = \exp(2\pi i/4)$

Example: E_7 gauge group: $y^2 = x^3 + z^3x$ “Type III^* ”

Example: $SU(2)$ gauge group: $y^2 = x^3 + zx$ “Type III ”

$$\Omega_1^{GL^+(2,\mathbb{Z})}$$

$$\Omega_1^{GL^+(2,\mathbb{Z})} = \mathbb{Z}_2 \times \mathbb{Z}_2$$

STILL have 7-branes with $\tau = \exp(2\pi i/4)$

DON'T have 7-branes with $\tau = \exp(2\pi i/3)$

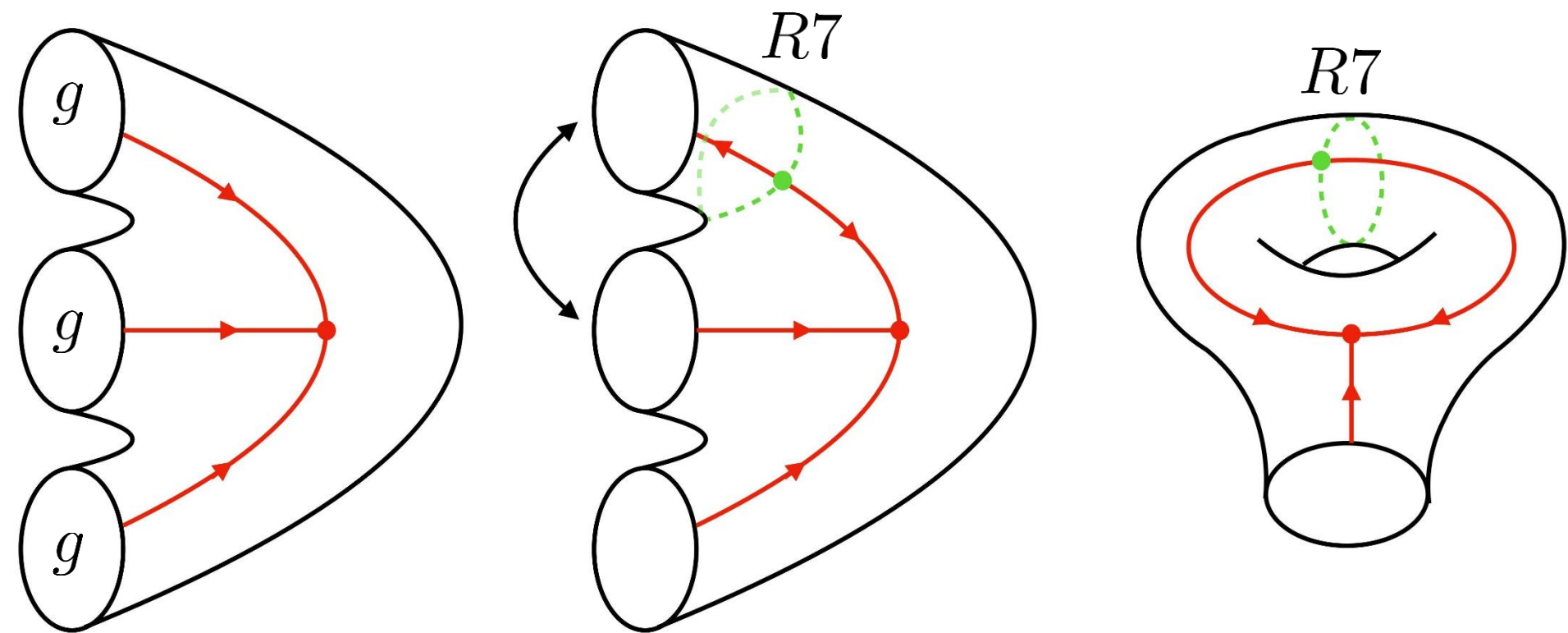


A NEW \mathbb{Z}_2 Class: “Reflection 7-Brane”




$$\Omega_1^{Mp(2,\mathbb{Z})} \quad \text{vs} \quad \Omega_1^{GL^+(2,\mathbb{Z})}$$

Eating \mathbb{Z}_3 with an R7 ($g^3 = 1$ in \mathbb{Z}_3)



R7-Branes

$GL(2, \mathbb{Z})$ Monodromy

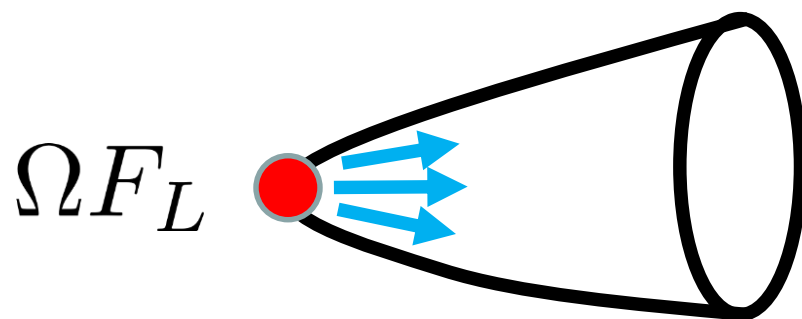
S-Dual 

$$M_{\Omega} = \begin{bmatrix} +1 & 0 \\ 0 & -1 \end{bmatrix}$$
$$M_{F_L} = \begin{bmatrix} -1 & 0 \\ 0 & +1 \end{bmatrix}$$

$$M_{SO(8)} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

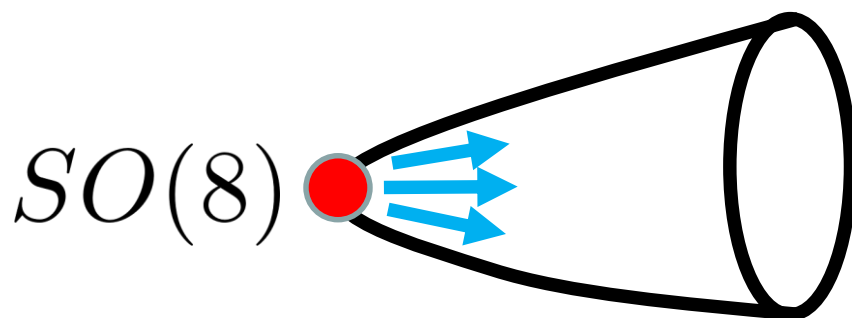
$$SO(8) \text{ 7-Brane: } y^2 = x^3 + \alpha z^2 x + z^3$$

ΩF_L Bound State



12

SAME τ Monodromy



~~SUSY~~

Ω and F_L Break SUSY (no Q 's preserved)

But $SO(8)$ 7-Brane (4 D7's and an $O7^-$)
Is Supersymmetric

$\Omega + F_L \rightarrow SO(8) + \text{radiation}$

Processes

$$\Omega + \Omega \rightarrow \text{radiation}$$

$$F_L + F_L \rightarrow \text{radiation}$$

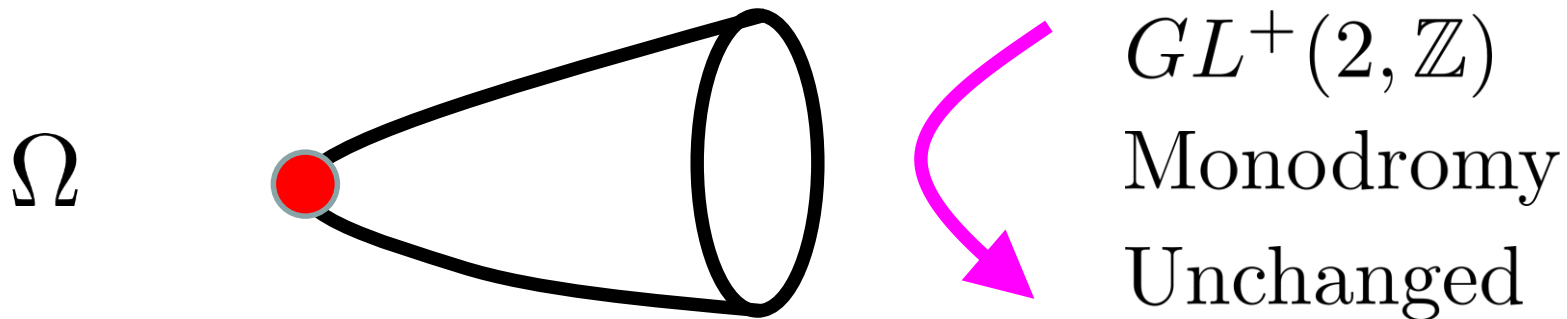
$$\Omega + F_L \rightarrow SO(8) + \text{radiation}$$

¿Stability?

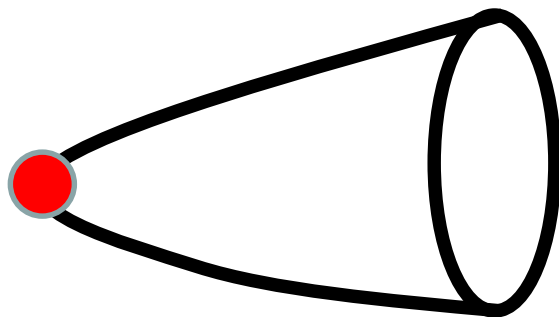
Not Required by Cobordism Conjecture...

Analogous to Type I D7-Brane (Unstable)

Sen '98



Ω

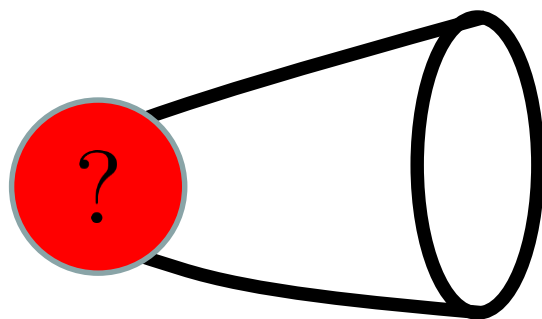


$GL^+(2, \mathbb{Z})$

Monodromy

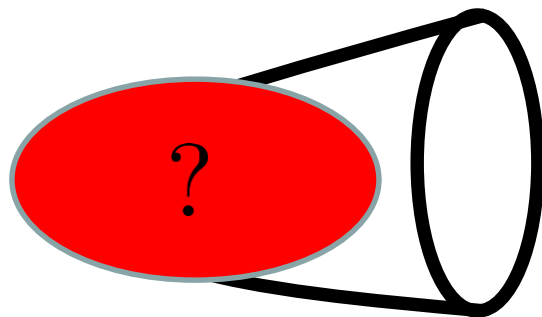
Unchanged

Ω



$GL^+(2, \mathbb{Z})$
Monodromy
Unchanged

Ω



$GL^+(2, \mathbb{Z})$

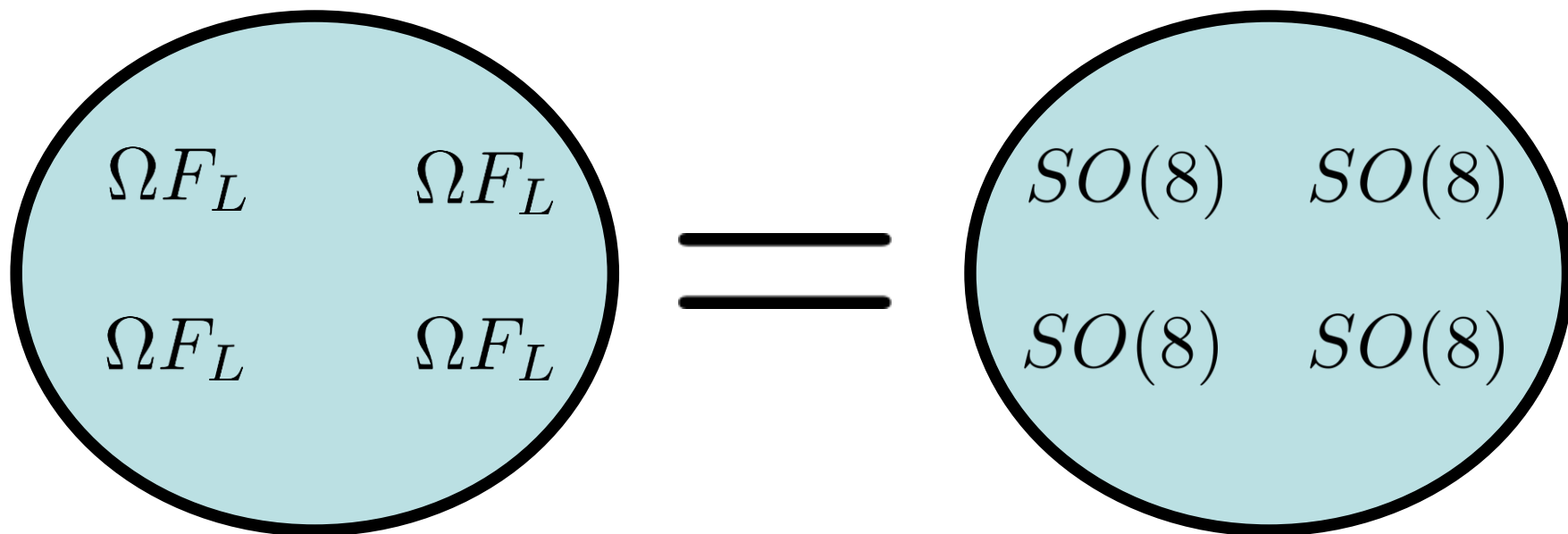
Monodromy

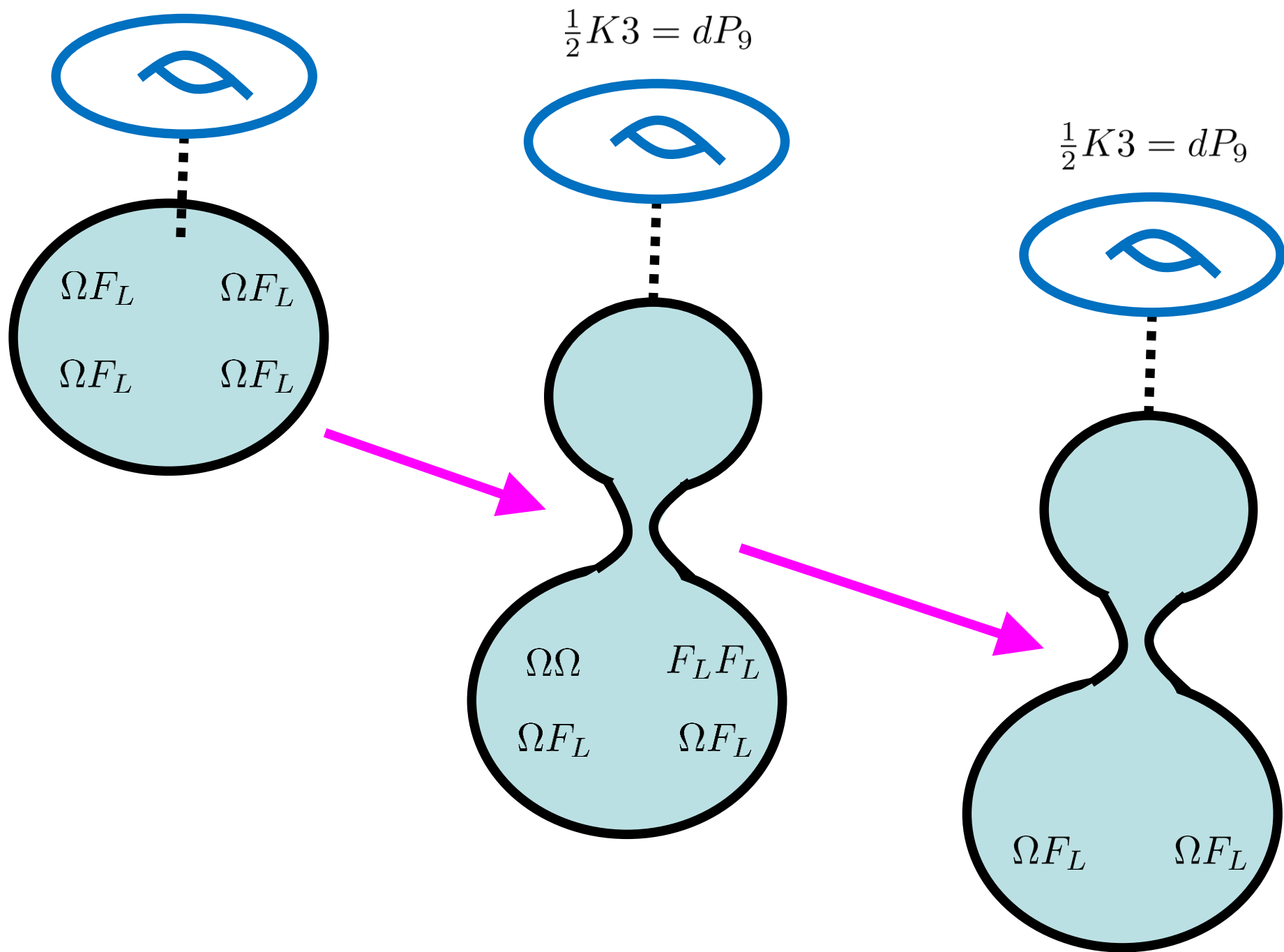
Unchanged

Rearrangements

Sen Limit: F-th on K3 with $SO(8)^4$

10D Spacetime: $\mathbb{R}^{7,1} \times \mathbb{CP}^1$





How About $\Omega_k^{\mathcal{G}}$?

$$\Omega_k^{Mp(2,\mathbb{Z})} \quad \text{and} \quad \Omega_k^{GL^+(2,\mathbb{Z})}$$

$\Omega_k^{Mp(2,\mathbb{Z})}$ for k odd all known SUSY bkgnds

$\Omega_k^{GL^+(2,\mathbb{Z})}$ for k odd: Just add R7's!

S-Folds and $\Omega_5^{Mp(2,\mathbb{Z})}$

S-Fold: Non-Perturbative Genⁿ of O3-Plane

$$\Omega_5^{Mp(2,\mathbb{Z})} = \mathbb{Z}_{32} \times \mathbb{Z}_2 \times \mathbb{Z}_9$$

F-theory Picture:

\mathbb{Z}_{32} via $\partial(\mathbb{C}^3 \times T^2)/\mathbb{Z}_4$ “Standard” Spin- \mathbb{Z}_8 -Bundle

\mathbb{Z}_2 via $\partial(\tilde{\mathbb{C}}^3 \times T^2)/\mathbb{Z}_4$ Different Spin- \mathbb{Z}_8 Bundle!

\mathbb{Z}_9 via $\partial(\mathbb{C}^3 \times T^2)/\mathbb{Z}_3$ “Standard” Spin- \mathbb{Z}_6 Bundle

4D SCFTs

Probe with N D3-Branes \Rightarrow SCFTs
(natural extra sectors)

\mathbb{Z}_{32} via $\partial(\mathbb{C}^3 \times T^2)/\mathbb{Z}_4$ “Standard” Spin- \mathbb{Z}_8 Bundle
 $\Rightarrow \mathcal{N} = 3$ SCFT

\mathbb{Z}_2 via $\partial(\tilde{\mathbb{C}}^3 \times T^2)/\mathbb{Z}_4$ Different Spin- \mathbb{Z}_8 Bundle!
 $\Rightarrow \mathcal{N} = 1$ SCFT

\mathbb{Z}_9 via $\partial(\mathbb{C}^3 \times T^2)/\mathbb{Z}_3$ “Standard” Spin- \mathbb{Z}_6 Bundle
 $\Rightarrow \mathcal{N} = 3$ SCFT

4D SCFTs

Probe with N D3-Branes \Rightarrow SCFTs
(natural extra sectors)

\mathbb{Z}_{32} via $\partial(\mathbb{C}^3 \times T^2)/\mathbb{Z}_4$ “Standard” Spin- \mathbb{Z}_8 Bundle
 $\Rightarrow \mathcal{N} = 3$ SCFT

\mathbb{Z}_2 via $\partial(\tilde{\mathbb{C}}^3 \times T^2)/\mathbb{Z}_4$ Different Spin- \mathbb{Z}_8 Bundle!
 $\Rightarrow \mathcal{N} = 1$ SCFT NEW SCFTs...

\mathbb{Z}_9 via $\partial(\mathbb{C}^3 \times T^2)/\mathbb{Z}_3$ “Standard” Spin- \mathbb{Z}_6 Bundle
 $\Rightarrow \mathcal{N} = 3$ SCFT

Summary / Future

Summary / Future

Summary:

- $\Omega_k^{\text{Duality}} + \text{Swamp} \Rightarrow \text{Predict Objects}$
- New Non-SUSY Ingredient: R7-Brane
- New 4D $\mathcal{N} = 1$ SCFTs

Future:

- ~~SUSY~~ and R7?
- Bigger Duality Groups, e.g. $E_{7(7)}(\mathbb{Z})$?
- Model Building with R7's?